PORTUGUESE STOCK MARKET: A LONG-MEMORY PROCESS?

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Abstract. This paper gives a basic overview of the various attempts at modelling stochastic processes for stock markets with a specific application to the Portuguese stock market data. Long-memory dependence in the stock prices would completely alter the data generation process and econometric models not considering the long-range dependence would exhibit poor forecasting abilities. The Hurst exponent is used to identify the presence of long-memory or fractal behaviour of the data generation process for the daily returns to ascertain if the process follows a fractional brownian motion. Detrended fluctuation analysis (DFA) using linear and quadratic trends and the Geweke Porter-Hudak methods are applied to detect the presence of long-memory or persistence. We find that the daily returns exhibit a small amount of long memory and that the quadratic trend used in the DFA overestimates the value of the Hurst exponent. These findings are corroborated by the use of the Geweke Porter-Hudak method wherein the Hurst exponent is close to the DFA using the linear trend.

Keywords: geometric brownian motion, Hurst Exponent, Long-Memory, Detrended Fluctuation Analysis, Geweke Porter-Hudak method, stable distributions.

PORTUGALIJOS AKCIJŲ RINKA: AR INERTIŠKAS KAINŲ KITIMAS?

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1. Introduction

The attempt to model the data generation process for financial data dates back to Bachelier (1900) wherein he attempted to model the French government bond and its futures. For further details on Bachelier’s work refer Voit (2005). Bachelier (1900) and Black Scholes (1973) are consistent with the Efficient Market Hypothesis, first formulated by Samuelson (1965) and Fama (1970). The efficient market is an idealized, complex system, wherein the essential information about a traded asset is instantaneously incorporated in its price. As a foremost implication, the serial correlation of the rate of return is zero for any short-time scale, so that the return time series are random walks.

Most of the empirical works carried during the 1960s support the random-walk hypothesis. Over the last 40 years, however, financial markets have witnessed significant changes, and data showed important discrepancies between the Bachelier model and real markets. After the collapse of the Bretton Woods system, the value of currencies, together with other financial prices, commodity prices, including oil, and land prices were displaying fluctuations of an order of magnitude never experienced before. By this time, the electronic revolution started to adapt to financial markets and capital facilitating global capital movements all around the globe. The volume of financial transactions has since overwhelmed current account transactions by a factor of several hundreds to 1.

There are some striking features that do not fit with the Geometric Brownian motion Bouchaud (2002). Empirical evidence from stock markets around the world shows that the returns do not follow a Gaussian distribution but are fat tailed and skewed. Also the volatility of the returns \( r(t) \) shows heteroskedasticity with periods of high volatility and low volatility. Also financial series often show long memory wherein hyperbolic decay rates of the autocorrelation function of the log price, at odds with the efficient market hypothesis, were first observed by Greene and Fielitz (1977) and Taylor (1986). The major implication is the possibility of earning speculative profits by means of remote information.

Second, contrary to the Gaussian model, the data generating process display fat tails. The presence of kurtosis suggests that rare events should not be assumed away when it comes to managing risk. These distributions can be accurately described by power-law distributions. The exponent corresponding to emerging markets can be less than two, in which case the variance diverges to infinity.

Moreover, periods of hectic activity and relatively quiescent ones coexist. Such a clustering in the volume of activity and volatility leads to a multifractal-like behavior of returns. The leverage effect, a negative correlation between (past) returns and (future) volatilities in turn leads to negative skewness in the distribution of returns.

In this paper we investigate the presence of some of these facts, with a special focus on long-term dependency. The intuition behind long memory is that the longest cycle in a sample will be proportional to the number of observations (Mandelbrot et al. 1997). There is no definitive conclusion about the existence of long memory in financial returns. Green and Fielitz (1977), Taylor (1986), Barkoulas et al. (2000), Taqqu et al. (1999), Ding et al. (2001), Sadique and Silvapulle (2001) and others claim that financial markets exhibit long memory. Other scholars do not reach a clear-cut conclusion, such as Lo (1999). However, in the post Bretton-Woods era the dominant view is that long-term dependence exists in liquid markets up to a lag of a ten-minute order.

In face of this evidence it is important to investigate different stochastic processes and fit statistical distributions that mimicked the actual data as close as possible.

The first step toward this is to identify whether the process exhibits long or short memory or the assumption that the data does not exhibit memory holds. The presence of memory will then dictate the choice of models used to forecast the underlying process.

The mathematical definition of a stationary process with long-memory or long-range dependence or persistence is given by its autocorrelation function \( \rho_k \) such that

\[
\lim_{k \to \infty} \rho_k = c k^{-\lambda/2}
\]

for some \( 0 < \lambda < 1 \) (Cowpertwaite and Metcalfe 2009: 160). For a long-memory process the autocorrelation function decays slowly at a hyperbolic rate as opposed to an exponential rate for a Brownian motion. This implies that the autocorrelations are not summable or in other words

\[
\sum_{k=-\infty}^{\infty} \rho_k = \infty
\]

The spectral density defined as:

\[
f(\omega) = \frac{1}{2\pi} \sum_{k=-\omega}^{\omega} \rho_k e^{ik\omega}
\]

in Fourier frequency \( \omega \) tends to infinity at zero frequency \( f(\omega) \to C \omega^{\lambda-1} \) as \( \omega \to 0 \). We use the Hurst (1951, 1955) exponent (H) to identify the presence or absence of memory. Hurst used the rescaled-range statistic over a period \( k \) and found it proportional to \( k^{\lambda} \) for some \( H > 1/2 \). The Hurst parameter is defined by \( H = 1 - \lambda / 2 \) and hence ranges from \( 1/2 < H < 1 \). If \( \lambda = 1 \Rightarrow H = 1/2 \) and the process is a Brownian motion with no long-range dependence.

The next section gives a succinct introduction to the various approaches used in literature and to justify the methodology adopted to identify the plausible data generation process. The empirical evidence follows with the methodology and results. The final section concludes.

2. Literature Survey

Two main approaches are used to fit models to financial time series like stock prices or options data.
1. Identifying the underlying distribution for the data generation process by calibrating the actual observations to Stable Distributions

2. Fitting econometric models like ARCH, GARCH, FARIMA, FIEGARCH based on the existence of memory in the evolution of prices. The existence of memory in the process is based on the value of the Hurst exponent.

**Stable Distributions**

The data generation process of the stock prices is assumed to be a random walk of size \( x_i \) \( \forall i = 1, 2, ..., n \) with \( n \) i.i.d. changes at each instant of time \( \delta t \). The position of the random walk in time \( n \delta t \) equals the sum of the \( n \) i.i.d. \( x_i \). Thus \( S_n = \sum_{i=1}^{n} x_i \). The simplest example is \( x_i = s; \forall i = 1, 2, ..., n \).

The question is what happens to the probability distribution of \( S_n \) as \( n \) increases? If the functional form of the density function is invariant under the summation then the distribution is classified as **stable**. Thus if \( x_i \) follows a normal distribution with mean \( \mu \) and variance \( \sigma^2 \), then \( S_n = \sum_{i=1}^{n} x_i \), follows a normal distribution with mean \( n\mu \) and variance \( n\sigma^2 \).

Are there distributions that are stable with finite moments? Khintchine and Lévy (1936) derived the general formula for the entire class of stable distributions. Lévy stable distributions lack closed form density functions except for normal, Cauchy or Lorentzian and Lévy-Smirnov distributions. These distributions can be easily expressed in terms of the characteristic function, which is the Fourier transform of the distribution function \( p(x) \) given by

\[
\phi_p(t) = E[e^{itx}] = \int p(x)e^{itx}dx.
\]

The general form of the characteristic function of stable distributions is given by

\[
\log \phi_p(t) = \begin{cases} 
-\alpha |t|^\alpha \left( \beta \text{sign}(t) \tan \frac{\pi \alpha}{2} + i\mu t \right), & \text{if } \alpha \neq 1, \\
-\sigma |t|^{\beta} \left( \frac{\pi}{2} \log(t) + i\mu t \right), & \text{if } \alpha = 1, \end{cases}
\]  

(2)

where index of stability (tail index, exponent or characteristic exponent) \( \alpha \) in \((0,2)\) (for \( \alpha > 2 \), \( p(X) < 0 \)), a skewness parameter \( \beta \) in \([-1, 1]\), a scale parameter \( \sigma > 0 \) and location parameter \( \mu \) in \( R \). For details refer Mantegna and Stanley (2004); Weron (2001):

- Lévy-Smirnov: \( \alpha = 1/2, \beta = 1 \);
- Cauchy or Lorentzian: \( \alpha = 1, \beta = 0 \);
- Normal or Gaussian: \( \alpha = 2, \beta = 0 \),

when \( \beta = 0 \), the distribution is symmetric about \( \mu \). The pth moment of a Lévy stable distribution is finite if \( p < \alpha \). Thus all Lévy stable distributions have infinite variance except the normal. This has implications for risk management as Value at Risk studies normally attempt to estimate the probability of loss beyond a certain number of standard deviations below the mean.

**Self-Similarity**

Since one is modelling the distribution of the returns, the analysis may be sensitive to the scaling of the time factor. The point to consider is whether the distribution of returns \( r(\Delta t) = \frac{P(i + \Delta t) - P(i)}{P(i)} \) is self-similar? In other words is the distribution of returns taken over different time intervals \((\Delta t = 1, 2, 5, 10 \text{ minutes, 1 hour, 1 day, 2 days etc.})\) different? Mantegna and Stanley (2004) show that non-Gaussian stable distributions are self-similar when appropriately scaled. The next question is to find the appropriate scaling factor that reflects self-similarity. The approach to finding the scaling factor is to find the probability of return to the origin; \( p(S_n) = 0 \) and show that the rescaled distribution

\[
\widetilde{S}_n = \frac{S_n}{n^{1/\alpha}} \quad \text{satisfies} \quad \int p(S_n) d\widetilde{S}_n = 1.
\]

Truncated Lévy Flight (follows follows Mantegna and Stanley (2004)).

When each step takes time that is proportional to its length it is termed as a random walk. However when each step takes the same time regardless of the length, the random walk is termed as flight. When the steps are distributed according to a Lévy process it is termed as Lévy Flights. Except for the Gaussian distribution which is a stable Lévy distribution and hence scalable having a finite variance, no other Lévy distribution has finite variance though all are stable and scalable. Student’s t distribution does not possess scaling properties but has finite variance. The only distribution that possesses a finite variance and scaling behaviour over a large range is the Truncated Lévy Flight defined by:

\[
p(x) = \begin{cases} 
0, & x > 1, \\
c p_L(x), & -1 \leq x \leq 1, \\
0, & x < -1, 
\end{cases}
\]

(3)

where \( pL(x) \) is a symmetric Lévy distribution and \( c \) is a normalising constant. Mantegna and Stanley (2004) show that TLF distribution converges to the gaussian for large values of \( n \) i.e.

\[
S_n = \sum_{i=1}^{n} x_i,
\]

Estimation of Tail Index \( \alpha \) (follows Weron (2001)). When \( \alpha < 2 \), the tails of the Lévy distribution are asymptotically equivalent to a Pareto law, i.e. if \( X \sim \text{Sa}(\sigma, \beta, \mu), \alpha < 2, \sigma = 1, \mu = 0 \), then \( x \rightarrow \infty \)

\[
p(X > x) = 1 - F(x) \rightarrow C_\alpha (1 + \beta)x^{-\alpha},
\]

\[
p(X < -x) = F(x) \rightarrow C_\alpha (1 - \beta)x^{-\alpha},
\]

where \( C_\alpha = \frac{\zeta}{\pi} \Gamma(\alpha) \sin \left( \frac{\pi \alpha}{2} \right) \).
Log-log linear regression

To estimate the tail index, a linear regression is fit to the dependent variable log(1 - F(x)), where F(x) is the cumulative density function of \( x > 0 \). The sample is ordered so that \( X_{(1)} \geq X_{(2)} \geq \ldots \geq X_{(n)} \), and the Hill estimator based on \( k \) largest order statistics is

\[
\alpha_{\text{Hill}}(k) = \frac{1}{k} \sum_{i=1}^{k} \log \left( \frac{X_{(i)}}{X_{(i+k)}} \right)^{-1}
\]

Weron (2001) finds that Hill estimator also over estimates the tail index parameter \( \alpha \) and one needs to use high frequency data for asset returns and analyse only the most outlying values to correctly estimate \( \alpha \).

Brownian and Fractional Brownian motion

A Fractional Brownian Motion (FBM) (Vasconcelos 2004), is a Gaussian process \( \{W_t(t), t > 0\} \) with zero mean and stationary increments whose variance and covariance are given by

\[
E[W_H^2(t)] = t^{2H}
\]

where \( 0 < H < 1 \). It is a self similar process \( W_H(\alpha t) = a^dW_H(t) \) \( \forall a > 0 \). The parameter \( H \) is called the self-similarity exponent or the Hurst exponent. For \( H = 1/2 \), the FBM reduces to the usual Brownian motion where increments \( \Delta W_t = W_t(t + \Delta t) - W_t(t) \) are i.i.d. when \( H \neq 1/2 \), increments \( \Delta W_t \) are known as fractional white noise displaying long-range correlation

\[
E[\Delta W_{t+k}\Delta W_t] = 2H(2H-1)k^{2H-1} \quad \text{for; } k \to \infty
\]

Processes with lower \( H \) have a greater volatility than those processes with a higher \( H \). Fractional Differencing: FARIMA \( (p,d,q) \) process \( \{x_t\} \), FARIMA \( (p,d,q) \) has the form \( \phi(L)(1 - L)^d x_t = \psi(L)w_t \) for some \( -1 < d < \frac{1}{2} \). We fit \( \phi(L)(1 - L)^d x_t = \psi(L)w_t \) where

\[
(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \ldots
\]

and \( L \) is the backward lag operator. The autocorrelation function \( \rho_k \) of a FARIMA \( (0,d,0) \) process tends to

\[
\frac{\Gamma(1-d)}{\Gamma(d)} k^{d-1} \quad \text{for large } n.
\]

8. The Hurst exponent \( H \) is obtained from the scaling behaviour of \( F_T \), \( F_T = C_H T^H \) where \( C_H \) is a constant independent of the time lag \( T \).

3. Methodology

We use two methods to estimate the long-range dependence in the daily returns of the PSI20.

1. Heuristic method called the Detrended Fluctuation analysis.

Detrended Fluctuation Analysis

The Hurst exponent was initially estimated by using the rescaled-range (RS) analysis of Hurst (1951). We adopt the Detrended Fluctuation Analysis (DFA) methodology as described in Peng et al. (1994), Moreira et al. (1994) and Vasconcelos (2004) for estimating the Hurst exponent. Costa and Vasconcelos (2003) find the DFA more reliable than the RS analysis for estimation of the Hurst exponent. Various studies have been carried out to estimate the Hurst exponent to determine the existence of fractional Brownian motion and multi fractality. Razdan (2001) finds that the Bombay stock exchange exhibits fractional Brownian motion with mono-fractality using the RS analysis. Da Silva et al. (2007) estimate the Hurst exponent for the Brazilian exchange rate market using the RS analysis find it close to 0.5 implying a Brownian motion.

1. Given a time series \( r(t), t = 1, 2, \ldots, T \) of say daily returns, obtain the cumulative time series \( X(t) \).
2. Compute the rescaled function \( F_T(\tau) \) for each \( \tau \).
3. Break \( X(t) \) into \( N \) non-overlapping intervals of equal length \( \tau \) where \( N = \left\lfloor \frac{T}{\tau} \right\rfloor \), where \( N \) is an integer.
4. For each of the intervals, fit a linear regression \( Y_{\tau}(t) = a_n + b_n \tau t + \epsilon_\tau \) \( t \in \tau \) where \( a_n \) and \( b_n \) are obtained from an OLS estimation procedure.
5. Compute the rescaled function \( F_T \) for each \( \tau \).
6. \( F_T = \frac{1}{S} \sqrt{ \frac{1}{T} \sum_{t=1}^{T} (X(t) - Y_{\tau}(t))^2 } \)
7. Repeat steps 3,4,5 for different values of \( \tau \) and obtain \( F_T \) for each \( \tau \).

8. The Hurst exponent \( H \) is obtained from the scaling behaviour of \( F_T(\tau) \), \( F_T = C_H T^H \) where \( C_H \) is a constant independent of the time lag \( \tau \).
9. Use OLS regression on the $\log [F_t] = \log [C_{tt}] + H \log [\tau]$ to obtain $H$.

To check for multi fractality modify step 6 to

$$F_t = \left( \frac{1}{N_t} \sum_{i=1}^{N_t} X(t) - Y_{i.t} (t) \right)^{1/2}.$$  

**Spectral Density using the GPH test**

A time series $Y_t = \{ y_t \}_{t=1}^N$ is said to be integrated of order $d$, signified as $I(d)$ if it has a stationary, invertible autoregressive moving average (ARMA) representation after applying the difference operator $(1 - L)^d$ where $L$ is the backward lag operator. The series is fractionally integrated when $d$ is not an integer Geweke and Porter-Hudak (1983) suggested a semi-parametric estimator of $d$ in the frequency domain. They consider the data generating process $(1 - L)^d y_t = z_t$ where $z_t \sim I(0)$.

Representing the process in frequency domain $f_y(\omega) = \left| 1 - \exp(-i \omega) \right|^{-2d} f_z(\omega)$ where $f_y(\omega)$ and $f_z(\omega)$ are spectral densities of $y_t$ and $z_t$, respectively. The spectral density of the fractionally integrated process $y_t$ is given by

$$f_y(\omega) = \left[ 4 \sin^2 \left( \frac{\omega_j}{2} \right) \right]^{-d} f_z(\omega_j). \tag{4}$$

for $j = 1, 2, ..., n_f$ where $\omega_j = \frac{2\pi j}{N}$ and $f_z(\omega)$ is the spectral density corresponding to $z_t$. The fractional difference parameter $d$ can be estimated using the regression:

$$\ln \{ f_y(\omega_j) \} = \beta - d \ln \left[ 4 \sin^2 \left( \frac{\omega_j}{2} \right) \right] + \epsilon_j, \tag{5}$$

where $\beta = \ln(f_z(0))$ and $\epsilon_j = \ln \left( \frac{f_y(\omega_j)}{f_z(0)} \right)$.

Geweke and Porter-Hudak (1983) showed that using a periodogram estimate, the least squares estimate of $d$ using the above regression is distributed in large samples if the number of observations $n_f(T) = T^\alpha$ with $0 < \alpha < 1$ as a normal distribution

$$\hat{d} \sim N \left( d, \frac{\pi^2}{6 \sum_{j=1}^{n_f} U_j - \bar{U}^2} \right),$$

where

$$U_j = \ln \left( 4 \sin^2 \left( \frac{\omega_j}{2} \right) \right) \quad \text{and} \quad \bar{U} = \frac{\sum_{j=1}^{n_f} U_j}{n_f}.$$

Under the null hypothesis of no long memory ($d = 0$), the t-statistic $t_{d=0} = \hat{d} \left( \frac{\pi^2}{6 \sum_{j=1}^{n_f} (U_j - \bar{U})^2} \right)^{-1/2}$ has a limiting normal distribution.

### 4. Results

**Data and Simulation Presentation**

The section begins with the presentation of the data and distribution of the returns. This is followed by the analysis of the results.

The maximum return we find is 13.3486% while the minimum return is -12.8696%. Figure 1A shows the daily movement of the PSI20 (top) along with the daily returns (middle) and the volatility (bottom) of the returns. Figure 1B shows the distribution of the actual returns with a normal distribution superimposed on it. The actual distribution has a greater kurtosis and fatter tails which the normal distribution is unable to capture. The normal distribution has been fit using the maximum likelihood method with the parameters equated to the mean and the variance of the actual distribution.

**Fig. 1.** PSI20: Level, Returns, Volatility and density of daily returns and estimated normal fit
**Detrended Fluctuation Analysis**

The modified DFA method is used to identify the possible multi-fractal behaviour of the returns data and to find the evolution of the Hurst exponent over time across different time horizons with a moving window of 1 day.

Whether this is consistently true over the entire range was checked using a moving window of 1 day with a period of 3 years (750 days) for estimating each exponent.

Figure 3 gives a graphic representation of the linear and quadratic trend polynomials fit using the entire range of data consisting of 4251 points. The difference between $X_t$ and $Y_t$ is used for $F_\tau$, where $X_t$ is the cumulative series of the daily returns minus the mean of the series of daily returns. Eventually the nature of the errors between the series $X_t$ and its fit $Y_t$ will determine the Hurst exponent. In our case we find that the quadratic trends tend to overestimate the Hurst exponent implying that the quadratic trend may not be the appropriate fit for the local trends.

![Linear Trend](image1)

$Y_\tau (t) = a_n + b_n t$

**Quadratic Trend**

$Y_\tau (t) = a_n + b_n t + c_n t^2$

Fig. 2. Hurst exponent with Linear and Quadratic Trends

![Cumulative Sum of daily returns minus mean daily returns with Linear and Quadratic Trends](image2)

**Linear Trend**

$Y_\tau (t) = a_n + b_n t$

**Quadratic Trend**

$Y_\tau (t) = a_n + b_n t + c_n t^2$

Fig. 3. Cumulative Sum of daily returns minus mean daily returns with Linear and Quadratic Trends

The quadratic trend tends to overestimate the exponent over the entire data set. When using the entire data set we obtain a single value of the exponent.
Figure 4 shows the movement of the Hurst exponent (y axis) over the same time periods (x axis) for 3500 periods with a moving window of 1 day. Initially we use 750 days to estimate the Hurst exponent and then advance one day till the end of the data period. We have used the linear and quadratic trends to estimate the exponent. The Hurst exponent (lower series) estimated using the linear trend exhibits a lower average value as opposed to the quadratic trend (higher series). Except for a small period where the values of the Hurst exponent exhibit opposite behaviour (fall for the quadratic trend and rise for the linear trend) they both exhibit similar behaviour. The Hurst exponent based on the quadratic trend lies completely above 0.5 implying an unequivocal long-range dependence in the daily returns while the exponent based on the linear trend shows some periods when the exponent falls below 0.5. This implies anti-persistence and a faster return to the original level.

Table 1. Summary of Hurst Coefficients over time

<table>
<thead>
<tr>
<th>Trend</th>
<th>mean</th>
<th>Std deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.5515</td>
<td>0.0757</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.7520</td>
<td>0.0718</td>
</tr>
</tbody>
</table>

**Spectral Density using the GPH test**

We see from Figure 5 that the autocorrelations and the partial autocorrelations do not decay exponentially but show some persistence even at lags close to 35.

Table 2. GPH Test for estimating fractional difference parameter $d$

|            | Estimate | Std Error | t-value | Pr(>|t|) |
|------------|----------|-----------|---------|---------|
| $\beta$   | -9.9135  | 0.0537    | -184.675| <2e–16  |
| $d$        | -0.0786  | 0.0137    | -5.747  | 1.04e–8 |

We have the following relationships $\lambda = 1 - 2d$ and $H = 1 - \lambda / 2$. Thus from Table 2 we infer

$$H = 1 - \frac{(1-2d)^2}{2} = 1 - \frac{(1-2\times0.0786)^2}{2} = 0.5786$$

This shows a small persistence in the behaviour of daily returns. The Hurst exponent estimated using the Geweke and Porter-Hudak method is in close approximation with the linear trend used to estimate the Hurst exponent using the detrended fluctuation analysis.

**Comparison with Other Studies**

Podobnik et al. (2006) use the linear trend to estimate the Hurst exponent. Using their terminology, the DFA follows a scaling law $F_\tau \propto \tau^H$ and find that all the indices for the ten transition economies in Europe exhibit power-law auto-correlations or in other words have a Hurst exponent different from 0.5 implying long-range dependence. Alvarez-Ramirez et al. (2008a) find the Hurst exponent varies substantially from persistence (above 0.5) to anti-persistence (below 0.5). They infer that the end of Bretton-Woods era in 1972 had a major impact on the efficiency of the markets wherein they use the Hurst exponent as a proxy for the market efficiency and conclude that markets became more efficient. Contrary results to Alvarez-Ramirez et al. (2008a) are obtained by Onali and Goddard (2009) where they find no evidence of long-range dependence in the returns of the Italian Mibtel index. Wang, Liu and Gu (2009) studies the improvement in efficiency of the Shenzen stock market using the multi-fractal DFA and find that the Hurst exponent falls consistently across time thus concluding that the market became more efficient over time. Cajueiro and Tabak (2007) investigate the long-range dependence of LIBOR interest rates on maturities of fixed income instruments for six countries. They find that the long-range dependence falls with increased maturity for four countries out of six and rises for the remaining two. Serlitis and Rosenberg (2007) estimated the Hurst exponent for the NYMEX futures and found the series to be anti-persistent and thus price corrections occur much faster. Since the futures prices are intricately linked to the spot and option prices, the anti persistence may be as a result of the movements in the other markets where in the traders move much faster to rebalance their positions. On the contrary, with crude oil prices, Alvarez-Ramirez et al. (2008b) find the existence of persistence or a Hurst exponent in the range of 0.6–0.7 implying that the spot markets take time to adjust to information in the short run.
5. Conclusion
We have used two approaches to investigate the presence of long-range dependence in the daily returns of the PSI20. The detrended fluctuation analysis, a heuristic approach with a linear and quadratic trends over a large range of the returns series exhibit long-range dependence with a Hurst exponent greater than 0.5. This implies that the market is slow to respond to the shocks on the whole. It depends to be seen if the Hurst exponents are different during the rise and the fall as normally markets are quick to fall but slow to rise. The quadratic trend in the DFA method tends to rise. The quadratic trend in the DFA method tends to slow to respond to the shocks on the whole. It depends to rise. The quadratic trend may not be an appropriate fit to be used in the DFA.

We have used another semi-parametric approach to corroborate our estimates of the Hurst exponent using the Geweke and Porter-Hudak method. We find that the Hurst exponent estimated using this approach is closer to the Geweke and Porter-Hudak method. We find that the Hurst exponent and may not be an appropriate fit.

We propose the use of Fractional GARCH models to estimate the differencing parameter d and their use for forecasting as opposed to the traditional GARCH models.

Although there is a vast amount of empirical findings dealing with the issue of long-term dependence, its underlying causes remain obscure. Anti-persistence can be more easily interpreted on the grounds, for instance, of a learning process leading to price overreactions that are immediately adjusted. Indeed, to the eye, short-memory processes appear indistinguishable from a white noise (Mandelbrot et al. 1997). Long memory, on the contrary, indicates the existence of importance pieces of information that are not immediately incorporated in the price.

This fact suggests that there can be sources of information easily captured by prices, while others do not. There are several reasons why this may occur. A fractal or a multi-fractal series suggests the action of interacting systems generating positive feedback. During ‘normal’ periods in which those systems operate rather independently, the ‘low-scale’ information that becomes operative is that giving rise to short memory. Suddenly, the high-range information dominates the markets and long-term dependence appears.

For instance, in the benchmark of the current financial crisis, few scholars call into question that the huge amount of liquidity created by expansive monetary policies applied in the US and the Euro zone gave rise to a house bubble. This process, which resembled that occurred in Japan in the mid eighties, was for a long time compatible with a good performance of financial markets, growth, trade and other macroeconomic variables. According to Kindleberger and Aliber (2005) a situation like this becomes unsustainable whenever the ratio of the price of urban land to wage rises above a threshold level. In such a case, market adjustments push down land prices, giving rise to a scarcity of liquidity. Financial markets then collide with the real side of the economy making valuable units of information that were not operative prior to the process of revulsion.

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References


Appendix: R Code

# code to calculate the Hurst Exponent using Detrended Fluctuation Analysis
# file name is psi20-close.txt
# it has 4 columns, dd mm yyyy cl for day, month, year and close price
file.choose()
pt20<-read.table("psi20-close.txt",header=T)
names(pt20)

dp <- length(cl)
dr = dp-1
retc <- seq(0,0,length.out=dr)
volretc <- seq(0,0,length.out=dr)
fdc <- seq(0,0,length.out=dr)

for (j in 1:dr)
{
  retc[j] = (cl[j+1]-cl[j])/cl[j]
fdc[j] = cl[j+1]-cl[j]
volretc[j] = retc[j]^2
}
dretc <- retc - mean(retc)
m1 <- mean(retc)
m2 <- sd(retc)

# drawing graphs from here
par(mfrow=c(3,1))
plot(cl,type="l",panel.first=grid())
plot(retc,type="l",panel.first=grid())
plot(volretc,type="l",col='green',panel.first=grid())

# obtain the cumulative sum of returns
# reduced by mean of returns
X <- seq(0,0,length.out=dr)
X <- cumsum(dretc)
S <- sd(dretc)

# Detrended fluctuation Analysis
DFA <- function(tau,X1)
{
  dr = length(X1)
  In = as.integer(dr/tau)
  Yn <- matrix(nr=In,nc=tau)
  for (i in 1:In)
  for (j in 1:tau)
  {
    Yn[i,j] = X1[(i-1)*tau+j]
  }

  coeffx <- matrix(nr=In,nc=2)
  tempy <- seq(0,0,length.out=dr)
  tempx <- seq(0,0,length.out=tau)
  for (i in 1:In)
  for (j in 1:tau)
  {
    tempy[j] = Yn[i,j]
    tempx[j] = j
  }
```r
y.ld <- lm(formula = tempy ~ tempx)
coeffx[i,1] = coef(y.ld)[1]
coeffx[i,2] = coef(y.ld)[2]
}

lYt = In*tau
Yt <- seq(0,0,length.out=lYt)
Xt <- seq(0,0,length.out=lYt)
for(i in 1:In)
  for(j in 1:tau)
  { 
    Yt[(i-1)*tau+j] = coeffx[i,1]+coeffx[i,2]*j
    Xt[(i-1)*tau+j] = X1[(i-1)*tau+j]
  }

XtYtSq <- sum((Xt-Yt)^2)
Ft <- sqrt((1/lYt)*XtYtSq)
return(Ft)
}

EH <- function(Ft,taui)
{
  hurst.ld <- lm(formula = log(Ft) ~ log(taui))
cCH = coef(hurst.ld)[1]
cHH = coef(hurst.ld)[2]
return(cHH)
}

pX = 4
lp = floor(length(X)/pX)
nX <- matrix(nr=lp,nc=pX)
dnX <- matrix(nr=lp,nc=pX)
cumdnX <- matrix(nr=lp,nc=pX)
FtA <- matrix(nr=(lp-2),nc=pX)
tauA <- matrix(nr=(lp-2),nc=pX)
HurstC <- seq(0,0,length.out=pX)
nS <- seq(0,0,length.out=pX)
for (i in 1:Ip)
  for (j in 1:pX)
  { 
    nX[i,j] = retc[(j-1)*lp+i]
  }
nXmu <- seq(0,0,length.out=lp)
X1 <- seq(0,0,length.out=lp)
for(j in 1:pX)
  { 
    nXmu[j] <- mean(nX[,j])
nS[j] <- sd(nX[,j])
  }

for (j in 1:pX)
  { 
    dnX[j] = nX[,j]-nXmu[j]
    for(i in 1:lp)
      { 
        taui[i] = i+2
        Ft[i] = DFA(taui[i],X1)
        Fs[i] <- Ft[i]/nS[j]
      }
    FtA[,j] <- Fs
    tauA[,j] <- taui
    HurstC[j] <- EH(Fs,taui)
  }

H <- seq(0,1,length.out=21)
Ch <- seq(0,0,length.out=21)
for (i in 1:21)
  Ch[i] = sqrt(2/(2*H[i]+1)+1/(H[i]+2)-2/(H[i]+1))
tauH1 <- seq(0,0,length.out=(lp-2))
tauH2 <- seq(0,0,length.out=(lp-2))
tauH3 <- seq(0,0,length.out=(lp-2))
for (i in 1:lp)
  { 
    par(mfrow=c(2,2))
    for (j in 1:4)
      { 
        plot(log10(tauH[i]),log10(FtA[,j]),type='l' ,col=' dark red' ,panel.first=grid())
        lines(log10(tauH[i]),log10(FH1),type='l' ,col='blue')
        lines(log10(tauH[i]),log10(FH2),type='l' ,col='dark green')
        lines(log10(tauH[i]),log10(FH3),type='l' ,col='orange')
      }
  }
```

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